期中补充题目

$\lim\limits_{x\to+\infty} x^2(n^{\frac{1}{x}}-n^{\frac{1}{x+1}})$

$x^2n^{\frac{1}{x+1}}(n^{\frac{1}{x}-\frac{1}{x+1}}-1)$

$=x^2n^{\frac{1}{x+1}}(e^{\frac{1}{x(x+1)}\ln n}-1)$

$\sim\dfrac{x^2}{x(x+1)}n^{\frac{1}{x+1}}\ln n\to\ln n(n\to+\infty)$

$(\dfrac{f(x_0+\frac{1}{n})}{f(x_0)})^n,n\to+\infty$

$a=e^{\ln a}$

$e^{n\ln\dfrac{f(x_0+\frac{1}{n})}{f(x_0)}}$

$e^{\dfrac{f(x_0+\frac{1}{n})-f(x_0)}{\frac{1}{n}}\dfrac{1}{f(x_0)}}=e^{\frac{f'(x_0)}{f(x_0)}}$

注：也可以看作$\ln f(x)$求导

设f在$[a,b]$上连续,在(a,b)上二阶可导，证明：存在$\eta\in(a,b)$,有：

$$f(b)+f(a)-2f(\frac{a+b}{2})=\frac{(b-a)^2}{4}f''(\eta)$$

设$k=\dfrac{f(b)+f(a)-2f(\dfrac{a+b}{2})}{\dfrac{(b-a)^2}{4}}$

$\Rightarrow f(b)+f(a)-2f(\dfrac{a+b}{2})-\dfrac{(b-a)^2}{4}k=0$

$F(x)=f(x)+f(a)-2f(\dfrac{x+a}{2})-\dfrac{k(x-a)^2}{4}$

$F(a)=F(b)=0\Rightarrow F'(x_1)=0$

$F'(x_1)=f'(x_1)-f'(\dfrac{x+a}{2})-k\dfrac{x_1-a}{2}=0$

$\Rightarrow k=\dfrac{f'(x_1)-f'(\dfrac{x_1+a}{2})}{\dfrac{x_1-a}{2}}=f''(\eta)(Lagrange)$
